

SU(5) Monopoles and the Dual Standard Model

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Abstract

We find the spectrum of magnetic monopoles produced in the symmetry breaking $SU(5) \rightarrow [SU(3) \times SU(2) \times U(1)']/Z_6$ by constructing classical bound states of the fundamental monopoles. The spectrum of monopoles is found to correspond to the spectrum of one family of standard model fermions and hence, is a starting point for constructing the dual standard model. At this level, however, there is an extra monopole state - the “diquark” monopole - with no corresponding standard model fermion. If the $SU(3)$ factor now breaks down to Z_3 , the monopoles with non-trivial $SU(3)$ charge get confined by strings in $SU(3)$ singlets. Another outcome of this symmetry breaking is that the diquark monopole becomes unstable (metastable) to fragmentation into fundamental monopoles and the one-one correspondence with the standard model fermions is restored. We discuss the fate of the monopoles if the $[SU(2) \times U(1)']/Z_2$ factor breaks down to $U(1)_Q$ by a Higgs mechanism as in the electroweak model. Here we find that monopoles that are misaligned with the vacuum get connected by strings even though the electroweak symmetry breaking does not admit topological strings. We discuss the lowest order quantum corrections to the mass spectrum of monopoles.

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I. INTRODUCTION

The idea that particles and solitons are two descriptions of the same entities has an irresistible appeal: it promises to unify seemingly distant aspects of field theories and also to resolve various mysterious relations that might hold in a description of particles or solitons alone. Research over the past several decades [1,2] and (more intensely) over the last few years [3] has supported the particle-soliton duality and it appears that there is some truth in this idea. At the same time, the research has focussed on hypothetical field theory models where exact results can be derived. In this paper, we wish to follow up on the ideas presented in [4] and consider how duality might be relevant to the world we know.

It is well known that a theory with spontaneously broken symmetry can lead to magnetic monopole solutions [5] provided the vacuum manifold of the theory contains incontractible two spheres. Specifically, if $G \longrightarrow K$ denotes the symmetry breaking pattern where the initial symmetry group G is assumed to be simply connected, the monopoles are classified by the incontractible paths in K . This yields the topological classification of magnetic monopoles which has been repeatedly used over the last two decades to classify magnetic monopoles in field theories [6]. In spite of the success and widespread use of the topological classification of magnetic monopoles, there are reasons to believe that a classification of monopoles exists [7] that is finer than the topological classification. In the finer classification the monopoles form representations of a certain dual symmetry group K^v which has been explicitly worked out in a number of cases [7,8]. A monopole that appears to be the sole member in a given topological class is actually an irreducible multiplet of K^v and the field theory of monopoles has a K^v gauge symmetry.

These considerations have immediate application in the study of $SU(5)$ monopoles where the symmetry breaking one considers is:

$$G = SU(5) \longrightarrow H = [SU(3) \times SU(2) \times U(1)'] / Z_6 .$$

The spectrum of *stable* monopoles resulting from this symmetry breaking has been found [4] to be in correspondence with the fermions of a single family of the standard model and the dual group H^v is locally isomorphic to H [8]. Hence the stable monopoles of the purely bosonic $SU(5)$ model may just be a different description of the fermionic particles that are the constituents of the world we know. Our hope is that the present direction of research might eventually lead to a “dual standard model” which would be an alternate description of particle physics and would be immediately relevant to the strongly coupled QCD sector. A comparison of some of the features of the standard model versus the dual standard model is made in Table I.

It is worthwhile clarifying our use of the word “dual”. What we have in mind is that the monopoles of the $SU(5)$ model are simply a different description of the *fermions* in the standard model. In other words, the standard model is an effective theory of fields that create and annihilate $SU(5)$ monopoles. This is analogous to the sine-Gordon and Thirring model equivalence.

TABLE I. A comparison of the standard and the dual standard model. (g_3 , g_2 and g_1 refer to the coupling constants of $SU(3)$, $SU(2)$ and $U(1)$ respectively and the tildes denote the same quantities in the dual representation.) Both descriptions have a strong coupling problem - the standard model has quark confinement, the dual standard model has electroweak magnetic screening as described in Sec. VI.

Standard Model	Dual Standard Model
$SU(3)$ $g_3 > 1$ Quark confinement	$\tilde{S}U(3)$ $\tilde{g}_3 < 1$ String confinement
$SU(2) \times U(1)$ $g_2, g_1 < 1$ Higgs screening	$\tilde{S}U(2) \times \tilde{U}(1)$ $\tilde{g}_2, \tilde{g}_1 > 1$ Electroweak magnetic screening

We begin in Sec. II by constructing the entire spectrum of stable magnetic monopoles in an $SU(5)$ model. Here we find the existence of monopoles that are in correspondence with a single family of fermions of the standard model; in addition we find an extra monopole that we call the “diquark” monopole since it is a bound state of two (quark like) fundamental monopoles with twice the color charge. The classical mass spectrum of the stable monopoles is also determined in the weak coupling limit. In Sec. III we qualitatively consider quantum corrections to the mass spectrum of the monopoles and provide some partial results.

The correspondence between monopoles and standard model fermions requires that colored monopoles be confined by strings just like colored quarks are confined by QCD strings [9]. In Sec. IV we discuss how the confinement of “colored” monopoles is achieved by further breaking the “dual color” symmetry. At this stage, as described in Sec. V, we find that the diquark monopole becomes unstable (metastable) to fragmentation into fundamental monopoles and the correspondence of stable monopoles and standard model fermions becomes one to one.

We discuss the effect of the “dual electroweak” symmetry breaking on the $SU(5)$ monopoles in Sec. VI. In particular, we have tried to resolve the issue of what happens to $SU(5)$ monopoles when the dual electroweak symmetry group breaks down to the dual electromagnetic group. Based on classical arguments we conclude that monopoles whose magnetic charge is not purely electromagnetic get connected by strings even though the vacuum manifold for the electroweak symmetry breaking has trivial first homotopy. We discuss the limitation of the classical calculations we have used in the context of constructing the dual standard model.

In Sec. VII we discuss the issue of spin of the monopoles. We propose a simple scheme based on work done in the 70’s [10,11] to convert the monopoles into fermions. This scheme generates extra light dyonic states which can be eliminated by introducing a θ term in the action.

We summarize our findings in Sec. VIII and discuss future directions.

II. SPECTRUM OF STABLE MONOPOLES

Consider the symmetry breaking

$$G \equiv SU(5) \longrightarrow [SU(3) \times SU(2) \times U(1)'] / Z_6 \equiv H_{SM} \quad (1)$$

The subscript SM on H denotes that this group is also the symmetry group of the Standard Model. The Z_6 factor is actually the direct product $Z_3 \times Z_2$ where Z_3 is the center of $SU(3)$ and Z_2 is the center of $SU(2)$. The symmetry breaking can be realized when a Higgs field (Φ) in the adjoint representation of $SU(5)$ acquires a vacuum expectation value (VEV). The Higgs potential for Φ can be written as

$$V(\Phi) = -m_1^2(\text{Tr}\Phi^2) + a(\text{Tr}\Phi^2)^2 + b\text{Tr}(\Phi^4) \quad (2)$$

and the expectation value of Φ can be taken to be

$$\langle \Phi \rangle = v_1 \text{diag}(2, 2, 2, -3, -3) \quad (3)$$

where, $v_1 = m_1 / \sqrt{60a + 14b}$. After symmetry breaking, the scalar field Φ can be decomposed into pieces that transform in the $(\mathbf{8}, \mathbf{1})$, $(\mathbf{1}, \mathbf{3})$ and $(\mathbf{1}, \mathbf{1})$ representations of $SU(3) \times SU(2)$ (for a review see [12]). The masses of these scalar fields are denoted by $\mu_8 = \sqrt{20b} v_1$, $\mu_3 = 2\mu_8$ and $\mu_0 = 2m_1$ respectively. We shall be interested in the case $\mu_0 \ll \mu_8$.

The topological classification of the monopoles in this symmetry breaking is based on the second homotopy group

$$\pi_2(SU(5)/[SU(3) \times SU(2) \times U(1)'] / Z_6) \cong \pi_1([SU(3) \times SU(2) \times U(1)'] / Z_6) . \quad (4)$$

Hence the topologically inequivalent monopoles in this case can be found by considering the incontractible closed paths in H . The set of incontractible paths is not empty since $U(1)'$ is not simply connected. Yet the path that lies entirely in $U(1)'$ does not yield the fundamental monopole - the monopole that has the smallest non-vanishing $U(1)'$ charge. This is because the center of $SU(3) \times SU(2)$ is given by the group elements,

$$\{\mathbf{1}_3, e^{i2\pi/3}\mathbf{1}_3, e^{i4\pi/3}\mathbf{1}_3\} \oplus \{\mathbf{1}_2, e^{i\pi}\mathbf{1}_2\}$$

which also belong to $U(1)'$. ($\mathbf{1}_n$ denotes the $n \times n$ unit matrix.) Hence a path that goes around the $U(1)'$ one sixth of the way can be closed off by elements of the $SU(3)$ and $SU(2)$ groups. Specifically, an incontractible path yielding the fundamental monopole can be written as:

$$g(s) = \exp[iM_1 s] , \quad s \in [0, 2\pi] \quad (5)$$

where,

$$M_1 = Q_3 + Q_2 + Q_1 , \quad (6)$$

with,

$$Q_3 = \text{diag}\left(-\frac{1}{3}, -\frac{1}{3}, +\frac{2}{3}, 0, 0\right) \quad (7)$$

$$Q_2 = \text{diag} \left(0, 0, 0, +\frac{1}{2}, -\frac{1}{2} \right) \quad (8)$$

$$Q_1 = \text{diag} \left(+\frac{1}{3}, +\frac{1}{3}, +\frac{1}{3}, -\frac{1}{2}, -\frac{1}{2} \right) . \quad (9)$$

We can explicitly obtain the charge on the fundamental monopole from (6):

$$M_1 = \text{diag}(0, 0, 1, 0, -1) . \quad (10)$$

A path that traverses the whole $U(1)'$ is given by

$$e^{i6Q_1 s} , \quad s \in [0, 2\pi]$$

and, in this sense, the fundamental monopole only traverses one sixth of the full $U(1)'$ orbit. So the fundamental monopole has $1/6$ of magnetic $U(1)'$ charge. Similarly, a path that traverses the whole $U(1)$ subgroup of $SU(2)$ generated by Q_2 is given by

$$e^{i2Q_2 s} , \quad s \in [0, 2\pi]$$

and so the fundamental monopole has $1/2$ of magnetic $SU(2)$ charge. By identical arguments we find that the fundamental monopole has $1/3$ of magnetic $SU(3)$ charge.

Note that as long as the group G in (1) is simply connected, the topological classification of monopoles is given by $\pi_1(H_{SM})$ and is independent of G itself. So the same spectrum of monopoles will be produced for any simply connected G that breaks down to H_{SM} .

It has been conjectured for some time now [7] that the topological classification of magnetic monopoles is not the entire story and a finer classification of magnetic monopoles exists. In this finer classification scheme, the magnetic monopoles form a representation of a dual group. The existence of multi-dimensional representations of the fundamental monopole, for example, means that this monopole should not be regarded as a single monopole but as a member of a multiplet containing several degrees of freedom. In the specific case of $SU(5)$ fundamental monopoles, this multiplet corresponds to six different ways of writing the charge on the monopole which correspond to the six different ways of aligning the monopole charge in internal space. This is seen by realizing that the diagonal $SU(3)$ generator in (7) can be written in three different ways by putting the $+2/3$ in one of three different positions and the $SU(2)$ generator in (8) can be written in two different ways by putting the $+1/2$ in one of two different positions. We will denote the three different $SU(3)$ generators by T_c where c can be *blue*, *green* or *red*, the two different $SU(2)$ generators by λ_+ and λ_- and the $U(1)$ generator by Y . The explicit matrices are:

$$T_b = \text{diag} \left(-\frac{1}{3}, -\frac{1}{3}, +\frac{2}{3}, 0, 0 \right) \quad (11)$$

$$T_g = \text{diag} \left(-\frac{1}{3}, +\frac{2}{3}, -\frac{1}{3}, 0, 0 \right) \quad (12)$$

$$T_r = \text{diag} \left(+\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, 0, 0 \right) \quad (13)$$

$$\lambda_+ = \text{diag} \left(0, 0, 0, +\frac{1}{2}, -\frac{1}{2} \right) \quad (14)$$

$$\lambda_- = \text{diag} \left(0, 0, 0, -\frac{1}{2}, +\frac{1}{2} \right) \quad (15)$$

$$Y = \text{diag} \left(+\frac{1}{3}, +\frac{1}{3}, +\frac{1}{3}, -\frac{1}{2}, -\frac{1}{2} \right) . \quad (16)$$

Note that

$$T_b + T_g + T_r = 0 \quad (17)$$

and

$$\lambda_+ + \lambda_- = 0 . \quad (18)$$

With this notation, the six different fundamental monopole states can be labeled by their $SU(3)$, $SU(2)$ and $U(1)'$ charges:

$$|c, s, 1 > , \quad c = b, g, r , \quad s = +, - \quad (19)$$

and the charge on the monopole is

$$Q_m = T_c + \lambda_s + Y . \quad (20)$$

It should be remembered that all these six monopoles are topologically equivalent. But we should still treat them as being distinct because when we start combining them to form higher winding monopoles, the stability of the resulting monopole depends crucially on which two monopoles we consider. In other words, the interaction energy of two monopoles - which is a gauge invariant quantity - distinguishes between these monopoles¹.

The existence of six different fundamental monopoles has an important consequence that was first utilized by Gardner and Harvey [13] in the context of $SU(5)$ monopoles. If one attempts to construct charge two magnetic monopoles by combining two identical charge one monopoles, one can show that the resulting configuration is unstable due to the Coulombic repulsion between the monopoles. (In the critical Bogomolnyi case [14], the configuration is neutrally stable.) However, one can still combine non-identical fundamental monopoles and hope to obtain a stable charge two monopole. Since we have six non-identical monopoles, we have the possibility of getting ${}^6C_2 = 15$ different charge two monopoles that are stable. But not all these 15 monopoles will be stable. To decide which of the 15 combinations will be stable, we follow [13] and construct the interaction energies of two monopoles whose charges are written as:

¹One can also consider other ways of distinguishing between the monopoles. For example, in the event that the monopoles enter a region with magnetic field, the trajectory that the monopoles follow will be different for the different monopoles.

$$M_1 = n_1 Y + n_3 \lambda_i + n_8 T_a \quad (21)$$

$$M_2 = n'_1 Y + n'_3 \lambda_j + n'_8 T_b \quad (22)$$

where the n_i and n'_i are some integers. At separations larger than the monopole core size, the monopoles interact via gauge boson and scalar exchange and the interaction energy is:

$$V(r) = \frac{1}{4\alpha r} [n_1 n'_1 \text{Tr}(Y^2)(1 - e^{-\mu_0 r}) + n_3 n'_3 \text{Tr}(\lambda_i \lambda_j)(1 - e^{-\mu_3 r}) + n_8 n'_8 \text{Tr}(T_a T_b)(1 - e^{-\mu_8 r})] \quad (23)$$

where α is the $SU(5)$ fine structure constant. At separations less than the core size, it is assumed that the monopoles interact like Bogomolnyi-Prasad-Sommerfield (BPS) [14] monopoles and hence the interaction potential is flat. If the core size is very small compared to the other length scales, we can approximate the true interaction potential by (23) all the way down to $r = 0$. The monopole resulting from the combination of the two monopoles will be classically stable provided $V(r)$ is in the shape of a potential well at $r = 0$. Therefore, since $V(r)$ goes to zero as r goes to infinity, quantum stability can only be obtained if $V(r)$ is negative at the origin². This reduces the problem to determining the monopoles for which

$$V(0) = \frac{1}{4\alpha} [n_1 n'_1 \text{Tr}(Y^2) \mu_0 + n_3 n'_3 \text{Tr}(\lambda_i \lambda_j) \mu_3 + n_8 n'_8 \text{Tr}(T_a T_b) \mu_8] < 0 . \quad (24)$$

Let us start by constructing $n = 2$ monopoles by combining two $n = 1$ monopoles. Then $n_i = 1 = n'_i$, and we use:

$$\begin{aligned} \text{Tr}(Y^2) &= 5/6 , \\ \text{Tr}(\lambda_i \lambda_j) &= +1/2 , \quad \text{if } i = j , \\ \text{Tr}(\lambda_i \lambda_j) &= -1/2 , \quad \text{if } i \neq j , \\ \text{Tr}(T_a T_b) &= 2/3 , \quad \text{if } a = b , \\ \text{Tr}(T_a T_b) &= -1/3 , \quad \text{if } a \neq b , \end{aligned}$$

to obtain

$$24\alpha V(0) = 5\mu_0 + 10\mu_8 , 5\mu_0 + 4\mu_8 , 5\mu_0 - 8\mu_8 , 5\mu_0 - 2\mu_8 , \quad (25)$$

where, the four cases correspond to $(i = j, a = b)$, $(i = j, a \neq b)$, $(i \neq j, a \neq b)$ and $(i \neq j, a = b)$. This means that there are two cases which give stable charge two monopoles for sufficiently small μ_0 and these correspond to

$$|a, +, 1\rangle |c, -, 1\rangle = | -d, 0, 2\rangle , \quad a \neq c , \quad (26)$$

²We are restricting ourselves to cases where there is classical stability and there is also a chance that the monopole will be quantum mechanically stable. Metastable states - states that can decay by quantum tunneling - are not being considered here.

and,

$$|c, +, 1 > |c, -, 1 > = |cc, 0, 2 > . \quad (27)$$

In the first case, we have used (17) to write $a + c$ as $-d$ where d is one of b, g, r . We have also used (18) to set the $SU(2)$ charge to zero. As examples of these monopoles, we could have a monopole in the state $|-b, 0, 2 >$ and another in the state $|bb, 0, 2 >$. The value of $V(0)$ obtained from (25) gives the binding energy of these winding two monopoles.

The charge on the $n = 2$ monopole $|-b, 0, 2 >$ is:

$$M_2 = -T_b + 2Y = \text{diag}(1, 1, 0, -1, -1) ,$$

while that on $|bb, 0, 2 >$ is:

$$M'_2 = 2T_b + 2Y = \text{diag}(0, 0, 2, -1, -1) .$$

The charge three monopoles can be obtained by combining a charge two and a fundamental monopole. The stability requirement forces the color combinations to be bgr and hence there are only two distinct charge three monopoles:

$$|0, s, 3 > , \quad s = \pm . \quad (28)$$

In this connection, note that the monopole $|aa, 0, 2 >$ cannot be combined with any fundamental monopole to yield a stable monopole. For example, if we tried to combine $|bb, 0, 2 >$ with $|r, +, 1 >$ to get $|bbr, +, 3 >$, this would be unstable to decay into $|br, 0, 2 > = |-g, 0, 2 >$ and $|b, +, 1 >$.

The charge on the $n = 3$ monopole $|0, +, 3 >$ is:

$$M_3 = \lambda_+ + 3Y = \text{diag}(1, 1, 1, -1, -2) .$$

The charge four monopoles can be obtained by combining two charge two monopoles and this leads to the following three stable monopoles:

$$|c, 0, 4 > , \quad c = b, g, r . \quad (29)$$

It can be checked that this charge four monopole is stable to decay into a charge one and a charge three monopole. Also note that the monopole $|aa, 0, 2 >$ does not combine in any way with the other monopoles to give another distinct charge four monopole which is stable. For example, the combination $|bb, 0, 2 > |gg, 0, 2 >$ is unstable to decay into $|bg, 0, 2 > + |bg, 0, 2 >$ and the combination $|bb, 0, 2 > |gr, 0, 2 >$ is equivalent to $|b, 0, 4 >$ which is included in (29).

The charge on the $n = 4$ monopole $|b, 0, 4 >$ is:

$$M_4 = T_b + 4Y = \text{diag}(1, 1, 2, -2, -2) .$$

Gardner and Harvey [13] showed that a charge five monopole is always unstable to fragmentation into a charge two plus a charge three monopole. This follows at once since the charge two and charge three monopoles only interact via the hypercharge (Y) sector in (23) and this is always positive.

This then leads us to consider charge six monopoles and there is only one of them and we can obtain it by combining two charge three monopoles:

$$|0, 0, 6 \rangle . \quad (30)$$

Once again the monopole $|aa, 0, 2 \rangle$ cannot be combined with a charge four monopole to give a stable charge six monopole. For example, $|bb, 0, 2 \rangle |r, 0, 4 \rangle$ is unstable to decay into $|br, 0, 2 \rangle = | -g, 0, 2 \rangle$ and $|b, 0, 4 \rangle$.

The charge on the $n = 6$ monopole $|0, 0, 6 \rangle$ is:

$$M_6 = 6Y = \text{diag}(2, 2, 2, -3, -3) .$$

One can easily show that all the monopoles with charge greater than six are unstable to fragmenting into a monopole of charge six and something else. This is because the only interaction of the charge six monopole is via the hypercharge sector and this contributes positively to (23).

This completes the classical stability analysis of combinations of $SU(5)$ monopoles. Our analysis has uncovered the spectrum of monopoles that have a chance of being quantum mechanically stable. That is, so far we have rejected all monopoles that are classically unstable, or, are classically stable but still unstable to quantum tunneling. The charge spectrum and degeneracy of such H_{SM} monopoles is displayed in Table II where we also tabulate the spectrum of known fermions and their degeneracy [4]. The two spectra show remarkable agreement and so we shall name each of the monopoles by the symbol for the corresponding standard model fermion but with a tilde. (For example, the monopole $|b, +, 1 \rangle$ will be denoted by \tilde{u}_L^b .) In addition to the monopoles in one-one correspondence with the standard model fermions, we have also uncovered three extra monopole states $|cc, 0, 2 \rangle$, $c = b, g, r$. These states are doubly charged under $SU(3)$ and $U(1)'$ and are $SU(2)$ singlets and so we shall call them “diquark monopoles” and denote them by \tilde{x}^c . No correspondingly charged fermions are known to occur in Nature; as we shall see in Sec. V, these monopoles will become unstable (metastable) once $SU(3)$ breaks down and the colored monopoles get confined. Also, if we assume that transitions within the same topological sector occur rapidly, \tilde{x}^c will quickly decay into \bar{d}_R since both are $n = 2$ monopoles and \tilde{x}^c is more massive than \bar{d}_R .

Note that the monopoles transform under $SU(3)$, $SU(2)$ and $U(1)'$ gauge transformations. By simply counting the degeneracy of the monopoles, it is clear that all the monopoles transform in the fundamental representations of the symmetry groups. This also applies to the diquarks and they should transform in the fundamental representation of $SU(3)$.

TABLE II. Charges on classically stable $SU(5)$ monopoles and on standard model fermions. Also shown are the monopole degeneracies d_m and the number of fermions with a given set of charges, d_f . The monopoles in the last row do not have any corresponding fermions in the standard model.

n	n_3	n_2	n_1	d_m		$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	d_f
+1	1/3	1/2	+1/6	6	$(u, d)_L$	1/3	1/2	+1/6	6
-2	1/3	0	-1/3	3	d_R	1/3	0	-1/3	3
-3	0	1/2	-1/2	2	$(\nu, e)_L$	0	1/2	-1/2	2
+4	1/3	0	+2/3	3	u_R	1/3	0	+2/3	3
-6	0	0	-1	1	e_R	0	0	-1	1
+2	2/3	0	+1/3	3	?				

In the weak coupling case, the monopoles are very heavy compared to the other mass scales in the problem (μ_0 and μ_8) and the classical result for the interaction energy given in (24) can be expected to be accurate. Table III shows the classical masses of the various monopole bound states.

TABLE III. Masses of stable $SU(5)$ monopoles in the weak coupling regime.

n	Mass
+1	M_1
-2	$2M_1 + (5\mu_0/24 - \mu_8/3)/\alpha$
-3	$3M_1 + (5\mu_0/8 - \mu_8/2)/\alpha$
+4	$4M_1 + (5\mu_0/4 - 3\mu_8/4)/\alpha$
-6	$6M_1 + (25\mu_0/8 - 5\mu_8/4)/\alpha$
+2	$2M_1 + (5\mu_0/24 - \mu_8/12)/\alpha$

As the couplings get somewhat larger (while still remaining in the weakly coupled regime), we can expect departures from the classical result for the interaction energy. The quantum corrections to the interaction energy can (in principle) be found by solving the

Schrodinger equation with a reduced mass for the interacting monopoles and the potential given in (23) together with quantum corrections. Our investigation of this issue is necessarily incomplete because of the difficulties encountered in quantizing monopoles. We now describe our calculations and partial results.

III. STABLE MONOPOLES: DESCRIPTION OF QUANTUM EFFECTS

We consider the problem of determining the quantum corrections to the classical values of the mass of the monopoles shown in Table III. The full problem involves the quantization of monopoles and is beyond our present reach. However, the problem of determining the quantum bound state energy for the potential $V(r)$ in (23) can still be treated analytically within certain approximations.

For monopole masses that are large compared to μ_0 and $\mu_3 = 2\mu_8$, the classical results (shown in Table III) should be quite accurate. The quantum correction to the classical masses involve three factors: (i) the mass of the fundamental monopole changes due to quantum corrections, (ii) the interaction potential gets quantum corrections, and, (iii) the quantum bound state has higher energy than the classical bound state. For example, the mass of the $n = 2$ monopole will be:

$$M_2 = 2M_1 + V(0) + \hbar\Delta(V(r), M_1)$$

where,

$$M_1 = M_{1,c} + \hbar\delta M_1$$

is the mass of the fundamental monopole with first order quantum corrections,

$$V(0) = V_0(0) + \hbar\delta V(0)$$

is the classical interaction potential at the origin, $V_0(0)$, plus its quantum correction, and, $\hbar\Delta(V(r), M_1)$ is the quantum energy of the bound state of two particles each of mass M_1 and interacting via the potential $V(r)$. To first order in \hbar , we can replace $\Delta(V(r), M_1)$ by $\Delta(V_0(r), M_{1,c})$. Then,

$$M_2 = 2M_{1,c} + V_0(r=0) + \hbar\Delta(V_0(r), M_{1,c}) + \hbar 2\delta M_1 + \hbar\delta V(r=0) .$$

In this equation for the $n = 2$ monopole mass, we know the first two terms and will find the third. As far as we know, magnetic monopoles have not been explicitly quantized and so the quantum correction to the mass of the fundamental monopole, $\hbar\delta M_1$, is not known. We shall not attempt to quantize the fundamental monopole here and neither shall we attempt to find the quantum corrections to the interaction potential $\hbar\delta V(r=0)$ (qualitatively discussed below).

We now turn to evaluating $\Delta(V_0(r), M_{1,c})$. This can be obtained by solving the Schrodinger equation

$$-\frac{1}{2\mu_r^2}\vec{\nabla}^2\Psi + V(r)\Psi = E\Psi \tag{31}$$

where μ_r is the reduced mass of the two interacting monopoles and the potential $V(r)$ is given in (23). This Schrodinger equation can easily be solved by numerical methods. Here, however, we describe an approximation that leads to an analytic solution.

At weak coupling ($\alpha \rightarrow 0$) we know the mass of the fundamental monopole to be

$$M_{1,c} = \frac{M_X}{\alpha} = \frac{4\pi}{g} \sqrt{\frac{25}{2}} v_1 .$$

And

$$\frac{M_{1,c}}{\mu_8} = \frac{M_X}{\alpha \mu_8} = \sqrt{\frac{5}{8b}} \frac{4\pi}{g}$$

where, g is the $SU(5)$ gauge coupling constant. In the weak coupling regime, M_1 is very much larger than μ_3 and μ_0 and the bound state wave-function can be thought of as being concentrated at $r = 0$ where $V(r)$ has a global minimum. At somewhat larger coupling, the bound state wave-function will be a little more spread out and will experience the interaction potential at larger r . If the distance scale of the spread of the wave-function is still small compared to the other length scales μ_3^{-1} and μ_0^{-1} , the bound state wave-function can be found by expanding the interaction potential $V(r)$ to linear order in r . Then the Schrodinger equation reduces to:

$$-\frac{1}{2\mu_r^2} \vec{\nabla}^2 \Psi + (a_0 + a_1 r) \Psi = E \Psi . \quad (32)$$

where we have written,

$$V(r) = a_0 + a_1 r .$$

Here $a_0 = O(\mu_8/\alpha)$ and $a_1 = O(\mu_8^2/\alpha)$. Adopting spherical symmetry, the solutions to (32) are known to be Airy functions and up to a normalization factor can be written as:

$$\Psi(r) = \frac{\text{Ai}(R - e)}{r} \quad (33)$$

where

$$R = (2\mu_r a_1)^{1/3} r$$

and

$$e = (2\mu_r a_1)^{1/3} \frac{(E - a_0)}{a_1} .$$

In solving the Schrodinger equation, we have imposed the boundary conditions that the wave-function is finite at $r = 0$ and vanishes at infinity. The lowest energy eigenvalue, E_0 , is given by the smallest (in absolute magnitude) root of the Airy function and we get:

$$E_0 = a_0 + 1.86 \left(\frac{a_1^2}{\mu_r} \right)^{1/3} \quad (34)$$

Hence:

$$\Delta(V_0, M_{1,c}) = 1.86 \left(\frac{a_1^2}{\mu_r} \right)^{1/3} . \quad (35)$$

with $\mu_r = M_{1,c}/2$ and a_0, a_1 can be obtained by Taylor expanding the potential in (23). Using the dimensional estimate $a_1 = O(\mu_8^2/\alpha)$ and $M_{1,c} = M_X/\alpha$ gives us

$$\Delta \sim \mu_8 \left(\frac{\mu_8}{\alpha M_x} \right)^{1/3}.$$

One can apply (34) to higher winding monopoles and find the corresponding Δ . In fact, it is not at all difficult to numerically solve the Schrodinger equation for the full potential $V_0(r)$ and obtain the energy eigenvalues. But until we can determine the other quantum corrections, namely δM_1 and $\delta V(r=0)$, this exercise cannot yield the desired quantum corrections to the monopole mass spectrum.

An estimate of the binding energy of higher winding monopoles does not need δM but still needs $\delta V(r=0)$. To find $\delta V(r=0)$, one might hope to draw inspiration from QED where vacuum polarization effects lead to a modification of the Coulomb interaction potential between two charges. Here the analogous process would involve Feynman diagrams with monopole loops. The calculation of such processes is beyond our reach though these can probably be ignored at weak coupling. One might also consider the effects of loops in the gauge and scalar particle propagators due to exchange of electrically charged particles. The contributions of such processes are higher order in coupling constants and, in any case, cannot shield the monopole charge since the charge is magnetic. But the processes might still yield a magnetic dipole interaction. We hope to consider this problem separately.

IV. $SU(3)$ BREAKING AND MONOPOLE CONFINEMENT

Consider the case when the symmetry of our model is broken down further when three $SU(3)$ adjoint fields acquire non-parallel VEVs:

$$[SU(3) \times SU(2) \times U(1)']/Z_6 \longrightarrow [SU(2) \times U(1)']/Z_2. \quad (36)$$

As the $[SU(2) \times U(1)']/Z_2$ factor is unaffected in this process, the symmetry breaking is effectively:

$$SU(3) \longrightarrow Z_3. \quad (37)$$

But now,

$$\pi_1 \left(\frac{SU(3)}{Z_3} \right) = Z_3 \quad (38)$$

and so the symmetry breaking yields Z_3 strings. These strings can terminate on monopoles since the full symmetry breaking

$$SU(5) \longrightarrow [SU(2) \times U(1)']/Z_2$$

does not yield any topological strings.

The monopoles \tilde{u}_L and \tilde{d}_L have $SU(3)$ charge equal to $1/3$. Indeed, from Table II, all the quark-monopoles have $1/3$ $SU(3)$ charge and so each of the quark-monopoles will get

connected to precisely one string. A string emanating from a monopole can then terminate on an anti-monopole or split into two strings (since they are Z_3 strings), with each of the two strings terminating on monopoles. An isolated system of monopoles and strings must necessarily be an $SU(3)$ singlet. A picture of a few possible monopole configurations is shown in Fig. 1 and these look exactly like the picture of mesons and hadrons emerging from the standard model.

The diquark monopoles have $SU(3)$ charge equal to $2/3$ and hence will get connected by two strings.

V. $SU(3)$ BREAKING AND THE MONOPOLE SPECTRUM

Once the colored monopoles get connected by Z_3 strings, the stability analysis of Sec. II does not apply. Instead the long range potential due to the exchange of colored gauge and scalar particles gets replaced by a linear string potential. Hence

$$\bar{V}(r) = \frac{1}{4\alpha r} [n_1 n'_1 \text{Tr}(Y^2)(1 - e^{-\mu_0 r}) + n_3 n'_3 \text{Tr}(\lambda_i \lambda_j)(1 - e^{-\mu_3 r})] - n_8 n'_8 \text{Tr}(T_a T_b) \alpha_s r \quad (39)$$

where, $\alpha_s > 0$ is a constant denoting the string tension. (Interacting monopoles having different $SU(3)$ charges are assumed to be connected to each other by strings while interacting monopoles with the same $SU(3)$ charges are taken to have strings pointing in opposite directions.) This expression for the potential is only valid on scales larger than the string thickness; on smaller scales, the expression in (23) should be valid.

As long as a monopole is constructed as a bound state of two differently colored monopoles, the confinement of colored monopoles does not affect the stability arguments of Sec. II. This means that the spectrum contains all the monopoles corresponding to the standard model fermions. But things are different for the diquark monopoles since these are constructed from two identically colored fundamental monopoles. Hence a diquark monopole must get connected by two strings and the long range interaction energy has the form:

$$V_{di}(r) = \frac{1}{4\alpha r} \left[\frac{5}{6}(1 - e^{-\mu_0 r}) - \frac{1}{2}(1 - e^{-\mu_3 r}) \right] - \frac{2}{3} \alpha_s r. \quad (40)$$

Therefore $V_{di}(r)$ is arbitrarily negative as r becomes large and the diquark monopole is unstable to fragmenting into its fundamental monopole constituents. (The Z_3 strings pull the diquark monopole apart into its quark-monopole constituents.) At short scales, however, we still expect that the interaction energy (23) is applicable and so there is a local minimum of the interaction energy at $r = 0$. This means that the diquark is not unstable classically; but it is metastable and can decay by quantum mechanical tunneling. This is in addition to the possible decay of \tilde{x}^c into \bar{d}_R as mentioned towards the end of Sec. II.

Hence we see that the symmetry breaking (36) can serve two purposes: the first is that it can confine the colored monopoles and the second is that it can make the diquark monopole unstable (metastable) and in this way restore the correspondence between the $SU(5)$ monopoles and the standard model fermions.

VI. $SU(2) \times U(1)$ SYMMETRY BREAKING BY A HIGGS MECHANISM

The results in Table III bear no resemblance to the mass spectrum of the standard model fermions. But there is no reason that we should expect any resemblance either. After all, the $SU(2) \times U(1)'$ is still unbroken - leading to identical masses for \tilde{u}_L and \tilde{d}_L , for example - while, in the standard model, the masses only arise from the electroweak symmetry breaking which also lifts the degeneracy between members of an $SU(2)$ doublet.

One way to lift the mass degeneracy between monopoles belonging to an $SU(2)$ doublet is to break the $SU(2) \times U(1)'$ symmetry further to $U(1)_Q$ via the Higgs mechanism, i.e. we consider:

$$G = SU(5) \longrightarrow H_1 = [SU(3) \times SU(2) \times U(1)'] / Z_6 \longrightarrow H_2 = [SU(3) \times U(1)_Q] / Z_3 \quad (41)$$

This symmetry breaking can be achieved if a field, χ , transforming in the fundamental representation of $SU(5)$ gets a VEV. The effect of the second symmetry breaking on the $SU(5)$ magnetic monopoles depends on which $U(1)$ subgroup of H_1 remains unbroken. The monopoles with dual electroweak charge M proportional to Q and quantized via the Dirac condition with respect to Q are “aligned” with the vacuum and remain unaffected by the second symmetry breaking. However, the monopoles for which the $SU(2) \times U(1)'$ charge is not proportional to Q are “misaligned” with the vacuum and cannot remain unaffected by the second symmetry breaking since their long range magnetic fields are not allowed in the H_2 vacuum. For example, let us look at the case that the residual $U(1)_Q$ is generated by (see eqns. (14) and (16))

$$Q = \lambda_- + Y = \text{diag} \left(+\frac{1}{3}, +\frac{1}{3}, +\frac{1}{3}, -1, 0 \right) .$$

The fundamental monopole states $|c, -, 1\rangle$, $c = b, g, r$ whose magnetic charges are $M = T_c + \lambda_- + Y$ are clearly aligned with Q while the monopoles $|c, +, 1\rangle$, $c = b, g, r$ with charges $M = T_c + \lambda_+ + Y$ are misaligned. The different fates of the aligned and misaligned monopoles during the second symmetry breaking causes the degeneracy between them to be lifted³.

The problem of determining the fate of misaligned monopoles has been considered by several authors in the past [15,16,13]. Two outcomes seem possible: (i) the misaligned monopoles get connected by strings, or, (ii) they rotate in internal space and get aligned. Both possibilities are difficult to reconcile with our general knowledge of topological defects. The first possibility is in doubt because the symmetry breaking (41) is known not to yield topological strings. And there is no known mechanism by which the second possibility can take place. A third possibility which one of us had envisioned before [4] was that the part of the magnetic field of the misaligned monopole that is not allowed in the H_2 phase gets screened just as the electric field of a charged particle gets screened in a Higgs phase. While

³ $SU(5)$ monopoles of winding greater than one are found to be always misaligned with the vacuum because the ratio of the dual hypercharge and $SU(2)$ charge that these monopoles carry differs from the ratio occurring in the generator Q of $U(1)_Q$.

this remains a possibility when quantum effects are taken into account, we will show that it is not allowed at the classical level.

The classic case of misalignment is that of the neutrino-monopole, $\tilde{\nu}_L$. We know that the neutrino has no long range electric field and hence, if the correspondence between monopoles and standard model fermions is to be true, we must have a monopole that has no long range magnetic field. At the classical level, this cannot be true as we now show in the subsection below. In the subsequent subsection, we construct an asymptotic solution of a misaligned monopole connected by a string.

A. Non-existence of a Classical “Neutrino” Monopole

Consider a Yang-Mills theory with a compact semi-simple group G undergoing two sequential symmetry breakings:

$$G \xrightarrow{\Phi} H_1 \xrightarrow{\chi} H_2$$

If Φ is in the adjoint representation, H_1 has the local structure of $U(1) \times K$ with $U(1)$ generated by $Q \propto \Phi$ and K some abelian or non-abelian group generated by K^α , $\alpha = 1, 2, \dots \text{Dim}(K)$. Although it is always possible to find K^α which locally satisfy $[K^\alpha, \Phi] = 0$ on the asymptotic two sphere, it is not generally possible to construct generators that are *globally* well-defined [17]. Hence, we will define the generators K^α in coordinate patches denoted by the letter P where $P = U$ means the “upper” patch (the whole two sphere except for a small region around the south pole) and $P = L$ means the “lower” patch (the whole two sphere except for a small region near the north pole). Then we can choose K_P^α such that:

$$\text{Tr}(\Phi K_P^\alpha) = 0, \quad \alpha = 1, 2, \dots \text{Dim}(K), \quad P = U, L. \quad (42)$$

Let $\Phi^0, W_{P\mu}^0$ be the scalar field and gauge potential of the monopole solution resulting from the first symmetry breaking. Asymptotically, they satisfy:

$$D_\mu \Phi^0 = \partial_\mu \Phi^0 - ig[W_{P\mu}^0, \Phi^0] = 0. \quad (43)$$

Now suppose after the second symmetry breaking the scalar field Φ and gauge potential W_μ become:

$$\Phi = \Phi^0 + \delta\Phi, \quad W_{P\mu} = W_{P\mu}^0 + \delta W_{P\mu} \quad (44)$$

We shall assume that Φ is in the same topological sector as Φ^0 and that it is possible to go to a gauge in which $\delta\Phi \propto \Phi^0$. In this gauge, it follows from (43) and

$$D_\mu \Phi = \partial_\mu \Phi - ig[W_\mu, \Phi] = 0$$

that δW_P can be expressed as:

$$\delta W_{P\mu} = A_\mu \Phi + B_{P\mu}^\alpha K_P^\alpha.$$

Note that A_μ is taken to be globally defined since the $U(1)$ generator Φ is globally defined. So now the field strength tensor has the form [18]:

$$G_{\mu\nu} = G_{\mu\nu}^0 + (\partial_\mu A_\nu - \partial_\nu A_\mu)\Phi + B_{P\mu\nu}^\alpha K_P^\alpha \quad (45)$$

where $G_{\mu\nu}^0$ is the field strength tensor of the monopole before the second symmetry breaking (constructed out of W_μ^0) and the structure of $B_{P\mu\nu}^\alpha$ is not important here. Now let us look at the $U(1)$ magnetic flux (denoted by h) through a sphere S_2 at infinity which contains the monopole:

$$h = \int_{S_2} d\vec{S} \cdot \text{Tr}(\Phi \vec{B})$$

where $B_i = \frac{1}{2}\epsilon_{ijk}G_{jk}$. From (42), (45) and Stokes theorem, we have:

$$h = \int_{S_2} d\vec{S} \cdot \text{Tr}(\Phi \vec{B}) = \int_{S_2} d\vec{S} \cdot \text{Tr}(\Phi \vec{B}^0) \quad (46)$$

where, $B_i^0 = \frac{1}{2}\epsilon_{ijk}G_{jk}^0$ and we are assuming that A_μ is not the gauge field for a Dirac monopole.

For the monopole to resemble a dualized neutrino after the second symmetry breaking, we would need $h = 0$. But this is not possible since the right-hand side of (46) is the $U(1)$ magnetic flux from the original monopole and is non-zero. Therefore a classical magnetic analog of the neutrino cannot exist.

If we allow ourselves the liberty of including Dirac monopoles, we could get $h = 0$ by cancelling the original monopole flux by an equal but opposite Dirac monopole flux in the Φ direction. But this configuration with $h = 0$ cannot be obtained by a continuous evolution of fields starting from the configuration of the original monopole. This is essentially because the Dirac monopole satisfies the discrete quantization condition and there is no way in which this discrete condition can be continuously relaxed.

The proof above can be summarized very simply: the monopole from the first symmetry breaking has a $U(1)$ magnetic flux which satisfies Maxwell's equations and hence cannot be screened.

B. Monopole Connected by a String

In this subsection we construct an asymptotic solution that describes a misaligned monopole connected by a string. For this purpose, it is cumbersome to consider the full $SU(5)$ model; instead we consider the simpler scheme

$$G = SU(3) \xrightarrow{\Phi} H = [SU(2) \times U(1)'] / Z_2 \xrightarrow{\chi} H' = U(1)_Q$$

where Φ and χ are in the adjoint (octet) and fundamental (triplet) representations of $SU(3)$. The form of the self-interaction for Φ is chosen so that its ground state is “ λ_8 like” (*i.e.* related to λ_8 by an $SU(3)$ rotation where $\lambda_a, a = 1, 2, \dots, 8$ are Gell-Mann matrices.) The fundamental monopole solution from the first stage has the configuration given by [19]:

$$\Phi^0 = \frac{1}{2}(\sqrt{3}\psi_1 - \psi_2) = \frac{1}{2\sqrt{3}}\text{diag}\left(1, \frac{3}{2}\vec{\sigma} \cdot \hat{r} - \frac{1}{2}\right) \quad (47)$$

$$W_\mu^0 = \frac{i}{e}[\psi_1, \partial_\mu \psi_1] = \frac{1}{2er}\hat{r} \times \vec{\lambda}' \quad (48)$$

where e is the gauge coupling and

$$\psi_1 = \frac{1}{2} \sum_i^3 \lambda'_i \hat{r}_i, \quad \psi_2 = \frac{1}{2} \lambda'_8, \quad \vec{\lambda}' = (\lambda'_1, \lambda'_2, \lambda'_3) \quad (49)$$

with λ'_i and λ'_8 defined by:

$$\vec{\lambda}' = \text{diag}(0, \vec{\sigma}), \quad \lambda'_8 = \frac{1}{\sqrt{3}} \text{diag}(-2, 1, 1) \quad (50)$$

and where $\vec{\sigma}$ are Pauli matrices. This configuration has the little group $H_{\hat{r}}$ which can be obtained from $H_{(0,0,1)} = H$ by conjugation: $H_{\hat{r}} = U(\hat{r}) H U^{-1}(\hat{r})$, where $U(\hat{r})$ is the $SU(3)$ transformation which relates $\Phi(\hat{z})$ and $\Phi(\hat{r})$:

$$U(\hat{r}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & -se^{-i\phi} \\ 0 & se^{i\phi} & c \end{pmatrix} \quad (51)$$

where $c = \cos \theta/2$, $s = \sin \theta/2$ and θ and ϕ are the usual spherical coordinates. The generators for infinitesimal transformations of the unbroken $SU(2) \times U(1)'$ can also be obtained by conjugation:

$$\tau_i(\hat{r}) = U(\hat{r}) \frac{1}{2} \lambda_i U^{-1}(\hat{r}), \quad i = 1, 2, 3$$

$$\Lambda = U(\hat{r}) \frac{1}{2} \lambda_8 U^{-1}(\hat{r}) = \Phi^0 .$$

(Note that here we are using the Gell-Mann matrices and not the primed versions defined in (50).) As discussed in [17], the construction of the off-diagonal $SU(2)$ generators works for all but one value of \hat{r} which we can choose to be on the negative z -axis. However, as we will never need to use the $SU(2)$ generators globally, this will not cause us any problems.

In terms of the diagonal generators of the little group, the magnetic charge on the monopole can be expressed as:

$$Q_M = -\frac{1}{e} \psi_1 = \frac{1}{2e} (-\sqrt{3} \Lambda + \tau_3) . \quad (52)$$

Now when the field χ gets a VEV, the symmetry breaks down to $U(1)_Q$ with Q satisfying:

$$Q\chi = 0 .$$

In the vacuum sector with $\Phi \propto \lambda_8$, we will consider two possible VEVs for χ :

$$\chi \propto \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} . \quad (53)$$

In the monopole sector, the situation becomes more complicated. For simplicity, we only discuss two cases where, like in the vacuum sector, a globally defined charge Q expressible as a linear combination of the $SU(2)$ and $U(1)$ generators with constant coefficients is available.

Explicitly, the VEVs for χ can be achieved by an $SU(3)$ rotation of the VEV along the \hat{z} direction:

$$\chi(\hat{r}) = U(\hat{r})\chi(\hat{z})$$

where $\chi(\hat{z})$ takes value as in (53).

At this stage, the asymptotic equations of motion are:

$$D_\mu \Phi = \partial_\mu \Phi - ie[W_\mu, \Phi] = 0 \quad (54)$$

$$D_\mu \chi = \partial_\mu \chi - ieW_\mu \chi = 0 \quad (55)$$

$$D_\mu G^{\mu\nu} = 0 \quad (56)$$

where, $G^{\mu\nu}$ is the field strength for the gauge field corresponding to the unbroken symmetry. As in the last section, the scalar field Φ and gauge potential W_μ can be written as in (44) and we can take $\Phi = \Phi_0$ at infinity. We now discuss explicit solutions to the equations of motion for the aligned and misaligned monopoles which correspond to the first and second choice for $\chi(\hat{z})$ given in (53).

1. Aligned Monopole

Let us first consider

$$\chi \propto U(\hat{r}) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

and then Q is given by:

$$Q = \frac{1}{2}(-\sqrt{3}\Lambda + \tau_3)$$

which is the same as the charge on the monopole Q_M in eqn. (52). Hence the charge on the monopole is aligned with the vacuum. In this case, the asymptotic equations of motion (54), (55) and (56) are satisfied trivially with $\Phi = \Phi^0$ and $W_\mu = W_\mu^0$ and the monopole is unaffected by the second symmetry breaking.

2. Misaligned Monopole

To obtain a misaligned monopole, we would like an asymptotic configuration for χ such that the charge Q satisfying $Q\chi = 0$ is not proportional to the monopole charge Q_M given in (52). One such Q would be:

$$Q = \frac{1}{2}(\sqrt{3}\Lambda + \tau_3) .$$

It is easy to check that this is indeed the charge if $\chi(\hat{z})$ is chosen to be the second possibility in (53). The configuration for $\chi(\hat{r})$ can then be found by using the $SU(3)$ rotations $U(\hat{r})$:

$$\chi \propto U(\hat{r}) \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ c \\ se^{i\varphi} \end{pmatrix} \quad (57)$$

Already it is clear that a string must get attached to the misaligned monopole since χ is becoming multi-valued at the south pole ($\theta = \pi$). We now find the gauge fields corresponding to the χ configuration in (57).

The misalignment of the monopole means that equation (55) is not satisfied trivially with $\Phi = \Phi^0$ and $W_\mu = W_\mu^0$ as happens for the aligned monopole. Using (44) and (48), the χ equation of motion (55) now reads

$$\partial_\mu \chi - \frac{i}{2r} \hat{r} \times \vec{\lambda}' \chi - ie \delta W_\mu \chi = 0 \quad (58)$$

with χ given as in (57). The solution to eq. (58) and (54) can be readily found:

$$\delta W_\mu = \frac{1}{er} \tan \frac{\theta}{2} \hat{e}_\phi \psi_1 + a_\mu Q$$

where ψ_1 was given in (49), \hat{e}_ϕ is the unit vector in the φ direction and a_μ is an arbitrary four vector function. Notice that the gauge potential has developed a Dirac monopole type potential where there is a line singularity along the south pole.

The field strength tensor is now given by

$$\begin{aligned} G_{\mu\nu} &= \partial_\mu W_\nu - \partial_\nu W_\mu - ie[W_\mu, W_\nu] \\ &= 0 + \frac{1}{e} f_{\mu\nu} Q \\ f_{\mu\nu} &= \partial_\mu a_\nu - \partial_\nu a_\mu . \end{aligned}$$

Then the gauge field equations, (56), reduce to $U(1)_Q$ field equations,

$$\partial_\mu f^{\mu\nu} = 0$$

except at the south pole where there is a singularity.

The fact that the field strength $G_{\mu\nu}$ is proportional to Q shows that the only long range magnetic fields are those allowed by the vacuum. The other fields must be screened but at the cost of introducing a string which, in our case, is along the negative z -axis. For example, in the case that the original monopole had vanishing field strength in the Q direction, we could take $a_\mu = 0$ and the monopole long range field would be completely screened but for the string attached to it. In this case, the above solution (in an asymptotic region excluding the south pole) is pure gauge with

$$W_\mu = -\frac{i}{e} \partial_\mu U(\hat{r}) U^{-1}(\hat{r}) .$$

The presence of the string attached to misaligned monopoles seems to be forced on us by the presence of the $SU(3)$ monopole even though the string is not topological. This might lead us to think that the string can also terminate on something else besides an $SU(3)$ antimonopole. An obvious candidate for the terminus of the string is Nambu's electroweak monopole [20].

Imagine the string connected to the $SU(3)$ monopole to be very long and along the $-z$ axis. In the region far from the $SU(3)$ monopole and in the vicinity of the string we have

$$\Phi = \frac{1}{2\sqrt{3}} \text{diag}(1, -2, 1) .$$

In this region, the string is like an electroweak Z -string [21] and can terminate on an electroweak magnetic monopole [20]. The χ configuration of Nambu's electroweak monopole is purely in the unbroken $SU(2)$ sector and is given by:

$$\chi = \begin{pmatrix} \sin(\theta'/2) \\ 0 \\ \cos(\theta'/2)e^{i\varphi} \end{pmatrix} \quad (59)$$

where, θ' is the spherical azimuthal angle measured from the $+z$ axis and with Nambu's monopole as the origin.

It would be satisfying to construct a globally well-defined configuration that represents an $SU(3)$ monopole connected by a Z -string to an electroweak monopole. However, we have not succeeded in constructing such a configuration. The problem is that we need to match the configurations in (57) and (59) in a smooth manner. However, the two configurations involve different components of χ and this makes the matching difficult. If a smooth matching turns out to be impossible, it would suggest that the strings connecting misaligned monopoles should be considered to be of topological origin within the low energy theory whose symmetry group is H in the one monopole sector. In other words, the string connecting the $SU(3)$ monopole would be unstable *only* to breaking via the formation of an $SU(3)$ monopole-antimonopole pair.

C. Resort to Strong Coupling?

Our conclusion then suggests that the correspondence between $SU(5)$ monopoles and standard model fermions fails at weak dual electroweak coupling within the realm of classical considerations. This, however, is not fatal to the correspondence because we know that the electroweak model is weakly coupled and hence the dual electroweak model must be strongly coupled (see Table I). So as far as the construction of the dual standard model is concerned, we must necessarily include strong coupling quantum effects in the dual electroweak symmetry breaking and the classical analyses described above may not be indicative of the true picture. While an analysis of strong coupling effects in the dual electroweak sector is beyond the scope of the present paper, we would like to mention that the considerations in [22] seem to be relevant. We hope to return to this issue some time in the future.

VII. θ ANGLE, SPIN AND STATISTICS

It is well-known that the presence of isospinor bound states can lead to spin on a monopole [10] with the usual spin-statistics connection [11]. If the model leading to the monopole includes a θ term, the condition for determining whether the monopole (dyon) is a fermion or a boson is simply [23] that if

$$\Sigma \equiv \frac{(mq - 2\theta)}{4\pi} \quad (60)$$

is half-integral (integral) then the dyon is a fermion (boson). (m is the magnetic charge and q is the total electric charge on the dyon.) Note that q contains the θ contribution to the electric charge of the dyon [24]. Since this contribution is $e\theta/2\pi$, where e is the electric charge of the adjoint field, the θ contributions cancel in (60) and so the statistics of the dyon does not depend on θ .

If we include an $SU(5)$ fundamental scalar field χ , we would expect that bound states of χ with the monopoles will convert the monopoles to dyons with spin half integral. For illustrative purpose, let us look at a simpler example - the 't Hooft-Polyakov monopole with an isospinor field U along the lines of Ref. [10]. In the presence of the field U , the two fundamental monopoles (monopole and antimonopole) give rise to a tower of dyonic states in which the smallest electric charge is $1/2$. The smaller charge states are shown in Fig. 2b and these include the two monopole states with zero electric charge and zero spin and four dyonic states with electric charge $\pm 1/2$ and spin $1/2$. The four spin- $1/2$ dyonic states can be written as $(\pm 1, \pm 1/2)$ where the first component is the magnetic charge and the second the electric charge, and they are degenerate in mass. This situation is unsatisfactory since the states with the lowest energy are bosonic, and even if we identify, say, the spin- $1/2$ $(+1, +1/2)$ dyon with u_L , the $(-1, -1/2)$ state would be \bar{u}_L but then there would be two other dyonic states, $\pm(1, -1/2)$, with the same mass that are not seen in Nature. The quadruple mass degeneracy is actually a consequence of P and CP invariance since a parity transform of $(+1, +1/2)$ yields $(-1, +1/2)$ and a CP transformation of $(+1, +1/2)$ yields $(+1, -1/2)$. Hence, it is clear that if we want to break the mass degeneracy of these four dyonic states, we must also break the P and CP connection between them.

A term that gives rise to P and CP violation is a θ term ($\theta \neq 0, \pi$). If such a term is included in the action, the quantization of the charge changes [24], and the lowest lying states become:

$$\begin{aligned} \vec{q}_{+0} &= \left(+1, \frac{\theta}{2\pi} \right) , \\ \vec{q}_{-0} &= \left(-1, -\frac{\theta}{2\pi} \right) , \\ \vec{q}_{++} &= \left(+1, +\frac{1}{2} + \frac{\theta}{2\pi} \right) , \\ \vec{q}_{+-} &= \left(+1, -\frac{1}{2} + \frac{\theta}{2\pi} \right) , \\ \vec{q}_{-+} &= \left(-1, +\frac{1}{2} - \frac{\theta}{2\pi} \right) , \\ \vec{q}_{--} &= \left(-1, -\frac{1}{2} - \frac{\theta}{2\pi} \right) . \end{aligned}$$

Now since the magnetic to electric charge ratios are identical for \vec{q}_{++} and \vec{q}_{--} but different from the ratio for \vec{q}_{+-} and \vec{q}_{-+} , we only expect two-fold degeneracy of the masses in fermionic states. An especially interesting case is when $\theta = \pi$ (see Fig. 2c). In this case, P and CP

violation are absent from the model since $\theta = \pi$ and $\theta = -\pi$ are related by a 2π shift. However, the four-fold degeneracy between the fermionic dyons is still broken and now the lightest dyonic states (of zero electric charge) are fermionic and only two-fold degenerate.

Note that in claiming that there is only two-fold degeneracy of the dyons, we are using the result that the dyons with smaller electric charge in Fig. 2 have smaller mass. This follows from the standard semiclassical quantization of monopoles. Suppose we consider a classical monopole configuration (including the U field) and we find that it has a mass M_0 where the subscript means that it has vanishing electric charge. Upon semi-classical quantization, to lowest order, the monopole's mass becomes:

$$M_q = M_0 + \frac{q^2}{2I} + \delta M, \quad (61)$$

where $q^2/2I$ comes from the quantization of the dyonic collective coordinate. Here $I > 0$ is a moment of inertia associated with the dyonic rotor degree of freedom. (For the quantization of the monopole see [26,27]; for reviews of the quantization procedure, see [6].) The δM term includes the zero-point energy from the quantization of small fluctuations around the original classical configuration and the possible contributions from the quantization of collective coordinates other than the dyonic rotor collective coordinate. These contributions only depend on the classical background that is being quantized and the values of the other quantum numbers - other than the electric charge - that the dyon might carry. So δM is the same for all the dyons that we are considering since these only differ in the value of their electric charge. Then from eq. (61) it is clear that the dyons with smaller electric charge in Fig. 2b have smaller mass.

To summarize this section, we can convert fundamental monopoles into fermions by considering isospin bound states on the monopoles that also convert them into dyons. This process leads to a four-fold mass degeneracy of dyons that can be reduced to a two-fold degeneracy by including a θ term in the action. In the special case when $\theta = \pi$, pure monopoles (with vanishing electric charge) can be fermions. This seems to us like the most attractive way for making the monopoles to be fermions as well as to only have two-fold (rather than four-fold) degeneracy⁴. A potential difficulty in this scenario - one that we are aware of - is that it may not be possible to get the higher charge monopoles to have spin 1/2 if the fundamental monopoles have spin 1/2 since the combination of two spin 1/2 particles is expected to yield an integral spin object. In future work we plan to confront this difficulty and to determine the spectrum of stable dyons in an $SU(5)$ model with a χ field.

VIII. CONCLUSIONS AND DISCUSSION

We have found the spectrum of stable $SU(5)$ monopoles and established a correspondence with a single family of fermions in the standard model. In addition to the monopoles in one-one correspondence with the standard model fermions, we find an additional monopole

⁴Note also that the stability of the higher winding monopoles as discussed in Sec. II is unaffected in this scenario since the electric charge vanishes.

which we have termed the “diquark” monopole. The charges and classical masses of the stable monopoles are displayed in Tables II and III.

Confinement of the colored monopoles is achieved by breaking the $SU(3)$ factor of the little group by a Higgs mechanism. This symmetry breaking also causes the diquark monopole to become unstable (metastable) and the correspondence between the charge spectra of monopoles and a single family of standard model fermions is exact. The picture of confined monopoles mimics the confinement of quarks as shown in Fig. 1.

At this stage, the masses of the monopoles occurring in an $SU(2)$ doublet are degenerate. To lift this degeneracy, it is necessary to break the electroweak factor $([SU(2) \times U(1)']/Z_2)$ of the symmetry group. We have investigated the consequences of achieving the symmetry breaking by a Higgs mechanism within the domain of classical physics. This has the undesirable effect that certain monopoles which are misaligned with the final vacuum get connected by strings. This is in spite of the fact that the electroweak symmetry breaking does not admit topological strings. Then for the correspondence between the monopoles and the standard model fermions to survive, we need to break the $SU(2) \times U(1)'$ factor by a mechanism other than the Higgs mechanism. An alternative may be to still use the Higgs mechanism to break the symmetry but, in addition, to consider a scheme which can screen the ensuing strings. Such phenomena are believed to occur in strongly coupled theories with some supporting evidence in supersymmetric theories [3]. The success of the monopole-fermion correspondence depends quite crucially on the determination of whether such a scheme exists in our context.

The undesirable consequence of the dual electroweak symmetry breaking on the misaligned monopoles is actually quite remarkable in light of particle physics as we know it. We know that at low energies, the electroweak coupling is weak and the strong ($SU(3)$) coupling is strong. This means that a dual model must have the reverse situation - the dual electroweak coupling must be strong and the dual strong coupling must be weak (see Table I). Then the strong coupling problem (quark confinement) that we encounter in the standard model must have a counterpart problem in the dual standard model. Furthermore, this problem must arise in the dual electroweak sector - just as we observe it to arise. This agreement between the two dual models seems to further support the idea that they are different descriptions of the same physics (*i.e.* two sides of the same coin).

The idea underlying our attempts to establish a monopole version of the standard model is that it may help us to understand various features of the standard model. These would include the charge spectrum of fermions and the representations in which the fermions occur. In addition, the model could be a natural description of the real world at strong coupling. Then the model would be useful to describe the confinement of quarks since this is a phenomenon associated with strong coupling in the standard model and hence weak coupling in the dual model. If the strong coupling aspects of the electroweak sector are easier to understand - perhaps in a lattice gauge theory context [22] - than strong coupling in the QCD sector, the dual standard model may yield a short cut to the simultaneous understanding of both the strongly and weakly coupled sectors in the standard model.

If the dual picture is correct, we can try and make predictions based on essentially classical considerations that may be experimentally testable. In the QCD sector, at least, the high energy scattering of particles should resemble the scattering of monopoles. Assuming that the results presently available for $SU(2)$ monopoles [28] also apply to monopoles in

other theories, quarks in head-on (zero angular momentum) collisions should scatter at 90° ; at larger impact parameters (*i.e.* larger angular momentum), the collisions should lead to products with greater transverse energies than predicted by point-like interactions. Another qualitative prediction of the $SU(5)$ model would be the existence of a metastable diquark.

Numerous issues remain to be resolved if we are to be able to think of standard model fermions as magnetic monopoles. These issues include determining the mass spectrum of the monopoles, getting the monopoles to have the right spin and statistics, introducing chirality in the monopoles and obtaining the family structure of the standard model [29]. At present, all these issues (and many more) are open for investigation.

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FIGURES

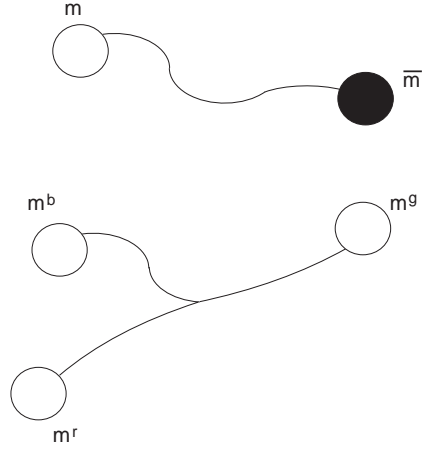


FIG. 1. Examples of confined clusters of monopoles

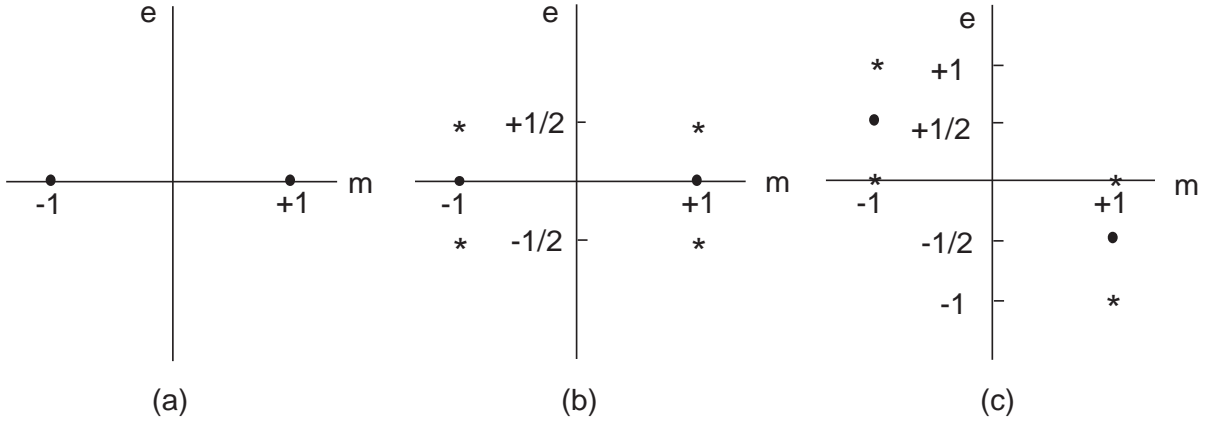


FIG. 2. The positions of fermionic (represented by \star) and bosonic (filled circles) dyons on the electric (e) and magnetic (m) charge plane. Fig. (a) shows the positions for two purely magnetic charges in the case when there are no electric bound states and $\theta = 0$. Fig. (b) shows the positions of the dyonic states when there are electric bound states with electric charge $1/2$. As shown in Fig. (c), on introducing $\theta = -\pi$, the positions of the fermionic and bosonic particles shift so that the purely magnetic and, presumably the lightest states, are fermionic. (The full spectrum of dyons will contain other states as well which we have not shown.)